MSEG 302 Spring 2018 Homework #10

**1. If the elastic modulus of cobalt (Co) is 200 GPa, and the elastic modulus of tungsten carbide (WC) is 700 GPa, calculate the upper and lower bound estimates of the elastic modulus of a 10 vol% Co – 90vol% WC composite.**

Per lecture and equations 16.1 and 16.2 in the text, Ec(upper) = EmVm + EpVp, and Ec(lower) = EmEp / (VmEp + VpEm). Here Vm = 0.1, Em = 200 GPa, and Vp = 0.9, Ep = 700 GPa. Substituting gives Ec(u) = 650 GPa, Ec(l) = 560 GPa.

**2. For a glass fiber-epoxy matrix composite, the critical ratio of fiber length to fiber diameter is 40. Using data in Table 16.4 in the text, determine the fiber-matrix bond strength.**

Equation 16.3 from text and lecture gives lc = f\* d/ (2 c), where lc is critical fiber length, d is fiber diameter, sf\* is tensile strength of fiber. So lc /d = sf\* / (2 tc). From Table 16.4, tensile strength of glass fiber is ~3.5 GPa. Solving for c with lc /d = 40 gives c = 44 MPa.

**3. For a continuous, fully oriented fiber-reinforced composite, the elastic modulus in the longitudinal direction is 33 GPa, while in the transverse direction it is 3.7 GPa. If the volume fraction of fibers is 0.30, determine the elastic moduli for the fiber and matrix phases.**

For continuous aligned in longitudinal direction (equation 16.10a)

Ecl = EmVm + EfVf = 33 GPa

For continuous aligned in the transverse direction (equation 16.15)

1/Ect = Vm/Em + Vf/Ef = 3.7 GPa

Two equations, two unknowns. Solving gives Ef = 104 GPa, Em = 2.6 GPa.

Mathematica:

Solve[33 GPa == emc vmc + efc vfc &&

1/(3.7 GPa) == vmc/emc + vfc/efc, {efc, emc}]

There seems to be another algebraic solution at Ef 1.2 GPa, Em = 46 GPa, but this is physically unrealistic, since fibers should be stiffer than matrix.

**4. Table 16.2 in the text gives data on the elastic modulus of glass-fiber reinforced polycarbonate composites. From these values, estimate the fiber efficiency parameter K for 20, 30, and 40 vol% fibers. Estimate the modulus of elasticity for a sample with 50 vol% fibers.**

From Table 16.2, for 20% Ec = 5.93 GPa, 30% Ec=8.62 GPa, and 40% Ec=11.6 GPa. Equation 16.20 gives Ecd = K EfVf + EmVm, where K is fiber efficiency factor. Modulus of unfilled polycarbonate is ~2.3 GPa (first column of Table 16.2), glass has estimated modulus of 73 GPa (Table 16.4). Solving for k gives:

Vf K

0.2 0.28

0.3 0.32

0.4 0.35

Mathematica:

kf = {{0.2, 0.28}, {0.3, 0.32}, {0.4, 0.35}}

ListPlot[kf, PlotRange -> {{0, 0.5}, {0, 0.4}}]

So plot of k vs. vf looks like:



So estimate of value k at vf = 0.5 is ~0.38. Using a fit to y = mx + b gives m=0.35, b=0.21, or k = 0.39.

Mathematica:

FindFit[kf, m x + b, {m, b}, x]

So. Ecd = (0.39) (0.5) (73 GPa) + 0.5 2.3 GPa = 15.4 GPa

Plot[0.35 x + 0.211, {x, 0, 0.5}]

Show[Out[n1], Out[n2]] (\*where n1,n2 are corresponding numbers of outputs from earlier Mathematica graphics function calls\*)

